GENERATING PRODUCTION PROFILES FOR AN OIL FIELD*

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Abstract. A zero-dimensional tank-type model for generating production profiles for an oil field is formulated and solved. The aggregation level of the model is discussed.

Keywords. Production profile; oil field; tank model.

INTRODUCTION

Models for generating production profiles from an oil-reservoir can be made at very different aggregation levels. One extreme solution is to use a very detailed reservoir simulator. This requires very detailed data, since there must be a consistency between the quality and amount of data and the level of disaggregation of the model. The running cost of such a model is considerable, and the number of computer runs will normally be very limited. The other extreme solution is to have a tank-type model from which any amount of oil can be extracted at any time as long as the "tank" is not empty. Such a model requires a minimum of data and is, of course, extremely cheap to run on a computer.

The choice of model will depend upon its intended use as well as the availability of data when the model is used. If the goal is to find the optimal production pattern for a field that has been in production for a while, it is natural to use a very detailed model. Not only will the quality and amount of data be good, but this type of model can answer a detailed question such as the one at hand, namely to find the optimal production rates from the individual wells. A tank-type model would not make much sense in this case.

If, on the other hand, the goal is to evaluate and rank a large number of fields for which very limited data are available (for example, in terms of expected net profit), a tank-type model might seem more suitable. When the government of a country, through the appropriate authorities, wants to allocate a number of blocks (via a concession round or a bidding process), it might require a consistent evaluation of the available blocks before it decides which blocks to offer in the next (concession or bidding) round. To do this evaluation, the only data available will often be from seismic investigations, and hence not very suitable for a detailed simulator. Also, the cost of using a detailed simulator on every potential field, would be enormous.

A third reason for needing production profiles is that one wants, on a rather aggregate level, to investigate the relationship among economic, geological and technical parameters that describe a field. Such an investigation is not done in order to say something about the development of a specific field, but to get a general feeling for how, for example, the geological and economic environments affect the optimal technical development plan.

The model presented in this paper is meant to be used for the latter two applications, see for example Nystad (1985a). It will be built on a relatively low number of variables, which are usually available at an early stage in the development of a field. In the next section of this paper we give a brief overview of the total model in which we use the profile generator described in the subsequent sections.

We wish to thank Dr. Joseph Warren for all his help with this paper, and Norsk Hydro A/S for financial support.

THE TOTAL MODEL

The reservoir model consists of several parts. The most important ones are the investment model for a platform, the production profile model, the tax model, and the unit for finding the (expected) net present value and optimal decisions for a field.

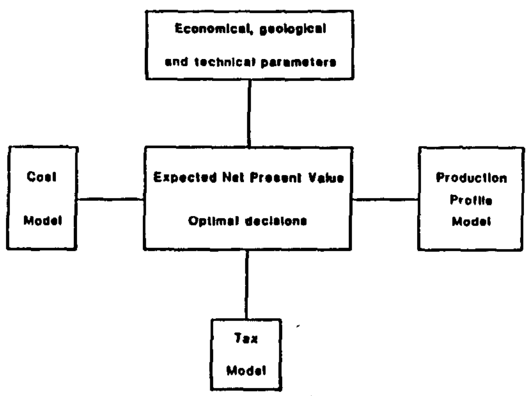

FIG. 1. General overview of the total model.

The cost model is built on the following principle. First the topside and substructure is split into approximately 15 modules. Based on the input data (production capacity of oil/gas, injection capacity of water, number of wells, water depth etc.) the total weight of each of these units is calculated. Then for each module we use an acquisition cost per ton and an installation cost per ton. The cost model also contains a unit for finding well-costs based on, for example, water depth.

The tax model contains a description of the Norwegian tax system. It makes it possible to analyse the economic effects of the taxation system. Of particular interest is seeing how the optimal decision changes because of the taxation system, i.e. Nystad (1985b).

The model can be used for both deterministic and stochastic analysis. Hence, the fact that the next section, which describes the profile generation model, is done in deterministic terms does not imply that it cannot be used in an uncertain environment.

THE PHYSICAL MODEL

In this section we describe a zero-dimensional tank-type model for generating production profiles for an oil field. First we give a simple physical model. The model is built on ideas similar to those in Warren (1978).

A. Physical Model Without Water-Injection

Let us first deal with the case without water-injection. Assume the following is true for the depletion of a closed reservoir.

$$Q = R\ (p_0 - p)/(p_0 - p_w) \qquad (1)$$

$$q' = q_i\ (p - p_w)/(p_0 - p_w) \qquad (2)$$

where

Q = accumulated production (MMbbl)
R = technically recoverable resources (MMbbl)
p = volumetrically weighted average reservoir pressure
p_0 = initial reservoir pressure
p_w = minimum (abandonment) pressure at well
q_i = initial field potential (MMbbl/year)
q' = field potential (MMbbl/year)

Formula (1) can also be written as $p = (p_0 - p_w)Q/R$ i.e., the pressure drops linearly with accumulated production. Formula (2) expresses how the potential of the field changes with pressure. At any point in time, we have

$q_i = q_i(t) = N(t)q_i'$, where

$N(t)$ = number of wells completed at time t (wells)
q_i' = initial well potential (MMbbl/year/well)

Eliminating p between (1) and (2), we get

$$q' = q_i\ (1 - Q/R) \qquad (3)$$

This shows how the field potential changes with accumulated production. Note that (3) is not a function of time.

The initial well potential q_i' is <u>not</u> equal to the production capacity of the well; in fact, we shall assume that the production capacity of a well is $\gamma q_i'$. This reflects that there are technical constraints on the production from a well. It is this difference that makes it possible to maintain a stable rate of production even after the last well has been completed. For details, see the next section.

B. Physical Model With Water Injection

In Section 3A above, we developed a model based on the pressure drop in the reservoir. Since the idea behind water injection is to keep the pressure up, the model cannot be used in that case. Therefore, consider the following very simple model.

In Figure 2 we show a tank-type model where the "tank" initially is filled with oil. As the oil is extracted from the reservoir, water is injected so that the pressure is maintained. We then have the following simple relation between accumulated production Q and remaining h/h_0, the relative part of the reservoir still filled with oil.

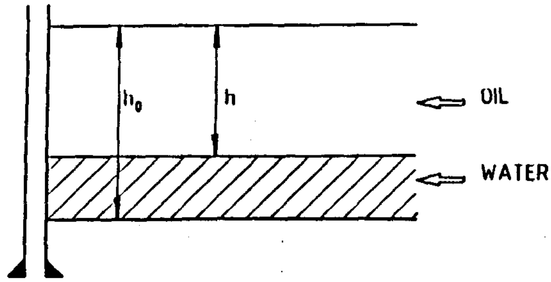

FIG. 2. A tank-type model for the case of water injection and pressure maintenance.

$$Q = R(h_0 - h)/h_0 \qquad (4)$$

Furthermore, we shall assume that

$$q' = q_i h/h_0 \qquad (5)$$

Solving (4) with respect to h and substituting into (5) gives:

$$q' = q_i\ (1 - Q/R)) \qquad (6)$$

Hence, we have the same model here as well. Comparing (3) and (6) one might think that we assume that water-injection has no effect. This, however, is not the case. The technically recoverable resources, variable R, will namely be larger for the water-injection case. We do not, however, have a model to describe this effect. A general belief seems to be that R will increase by between 50 and 100% in the case of full pressure maintenance.

GENERATING PROFILES

On the basis of the simple models presented in Section 3, we now show how production profiles can be generated. We shall assume the following input:

N - number of production wells to be drilled (wells)
ΔT - time span between completion of two production wells (years/well)
q_0 - the production capacity of the platform (MMbbl/year)
N_0 - number of predrilled wells (wells), $N_0 \leq N$
R - technically recoverable resources (MMbbl)
q_i' - initial well potential (MMbbl/year/well)
γ - maximal well production/initial well potential

Note that ΔT is not the time needed to complete on well, but rather the average time between completion of two <u>production</u> wells. Hence, if every second well is an injection well, ΔT is twice the time needed to complete one well.

There are four different cases to consider. In order to be able to distinguish between them, consider the following three variables:

t_N -$(N-N_0)\Delta T$: time at which drilling is finished

θ - $(\frac{q_0}{q_i'\gamma} - N_0)\Delta T$: time at which production equals the production capacity of the platform, provided decline does not occur earlier, and there are enough wells.

t_1 - time at which the production equals the potential, provided wells are drilled continuously until then, and provided the production capacity q_0 is so large that it is not constraining at any time.

Let $\tilde{N}(t)$ be the number of wells completed at time t, provided drilling is continued until then.

$$\tilde{N}(t) = (N_0 + \frac{t}{\Delta T})$$

Hence, the production capacity is $\tilde{N}(t)\, q_i^{\prime}\, \gamma$ whereas the field potential is equal to

$$\tilde{N}(t) q_i^{\prime} (1 - \frac{Q(t)}{R})$$

These two quantities are equal when

$$Q(t) = R\,(1-\gamma) \tag{7}$$

Hence N_1 (the number of producing wells completed when $Q(t) = R(1-\gamma)$) is given by

$$q_i^{\prime}\gamma\, (\frac{N_0+N_1}{2})\, (N_1 - N_0)\, \Delta T = R(1-\gamma)$$

which implies $N_1 = \sqrt{\dfrac{2R(1-\gamma)}{q_i^{\prime} \Delta T \gamma} + N_0^2}$

Therefore, $t_1 = (N_1 - N_0)\, \Delta T$

Now let us consider the three points in time, t_N, θ and t_1.

In the following, let $q(t)$ be the production rate at time t.

Case 1. $t_N < \theta$ $t_N \le t_1$. In this case, see Figure 3, we do not reach the production capacity of the platform, and decline does not occur before the last well has been completed.

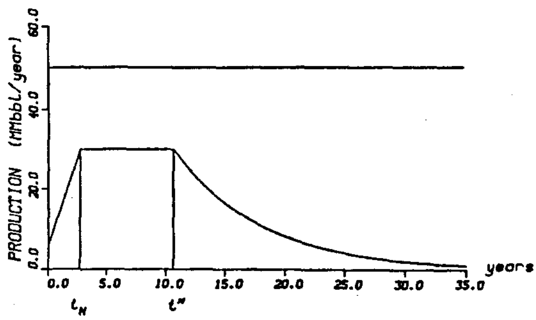

FIG. 3. Typical production profile for Case 1.

Case 2. $t_1 < t_N \le \theta$. We do not reach the production capacity of the platform. In addition, decline occurs before the wells are finished.

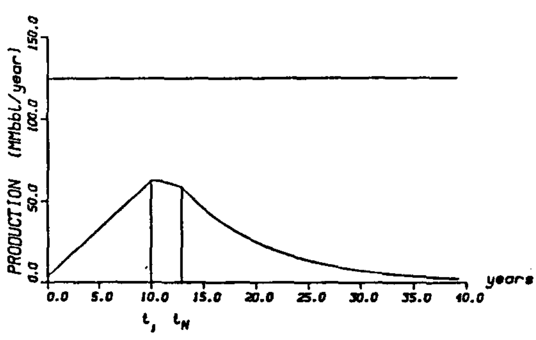

FIG. 4. Typical production profile for Case 2.

Case 3. $\theta \le \{t_1, t_N\}$. We reach the production capacity of the platform, q_0, without decline occurring. We have two cases here. Either decline occurs before t_N or after t_N. If it occurs after t_N, we have case 3a.

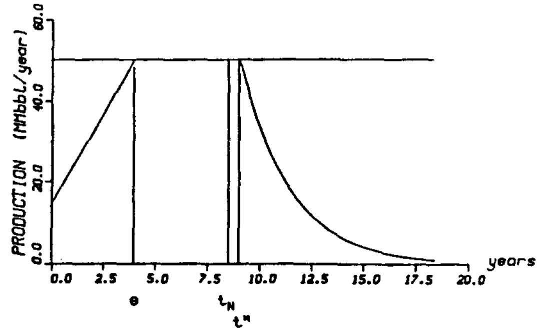

FIG. 5. Typical production profile for Case 3a.

If decline occurs before t_N, we have case 3b.

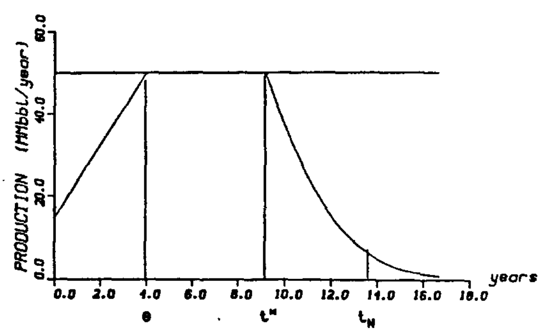

FIG. 6. Typical production profile for Case 3b.

Case 4. $t_1 < \theta < t_N$. In this case, the number of wells seemed to be large enough, but before q_0 was reached, the potential became lower than the production capacity of the wells; hence the production becomes equal to the potential. But even if the potential is lower than the well capacity, we might reach the production capacity of the platform. Therefore, we get Case 4a or 4b. In Case 4a, we behave as in Case 3a after q_0 has been reached.

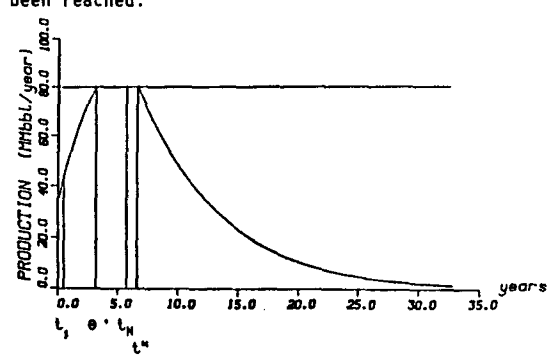

FIG. 7. Typical production profile for Case 4a.

In Case 4b, we behave as in Case 3b after q_0 has been reached.

If we do not reach q_0, Case 4 is equivalent to Case 2.

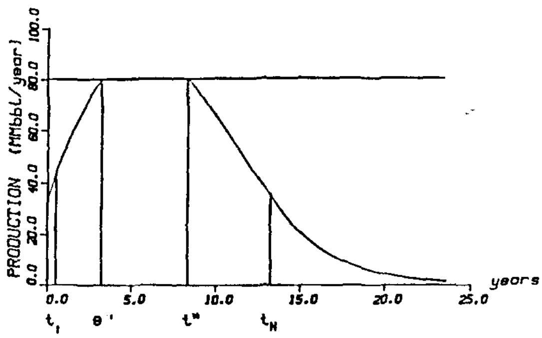

FIG. 8. Typical production profile for Case 4b.

Note that several details in Figures 3-8 have not yet been explained.

Case 1

Since $t_N < t_1$, we know that $Q(t_N) < R(1-\gamma)$. Once at the plateau, we will produce $N\ q_i^{\prime}\gamma$ until the field potential falls below this level. The field potential is equal to $N\ q_i^{\prime}\ (1-\frac{Q}{R})$. Hence, decline occurs when $Q^* = R(1-\gamma)$. The time t^* at which this occurs is found from

$$\frac{N_0+N}{2}\ q_i^{\prime}\gamma\ t_N + \gamma q_i^{\prime}\ N\ (t^*-t_N) = R(1-\gamma)$$

which implies $t^* = \frac{(1-\gamma)}{\gamma\lambda} + \frac{t_N}{2}\ (1-\frac{N_0}{N})$

where $\lambda = \frac{Nq_i^{\prime}}{R}$

Since after time t^*, the potential is lower than the capacity, the production will be equal to the potential.

In this case, $q(t) = \frac{d}{dt}\ Q(t)$, and in order to find $q(t)$, we must solve the differential equation:

$$\frac{d}{dt}\ Q(t) = N\ q_i^{\prime}\ (1 - \frac{Q(t)}{R}) \qquad t \geq t^*$$

with $Q(t^*) = R(1-\gamma)$

As a conclusion, we see that the production $q(t)$ is given by

$$q(t) = \begin{cases} (N_0 + \frac{t}{\Delta T})\ q_i^{\prime}\gamma & 0 \leq t \leq t_N \\ N\ q_i^{\prime}\ \gamma & t_N \leq t \leq t^* \\ Nq_i^{\prime}\gamma\ \exp\ (-\lambda(t-t^*)) & t \geq t^* \end{cases}$$

and

$$Q(t) = \begin{cases} q_i^{\prime}\gamma\ (N_0 + \frac{t}{2\Delta T})t & 0 \leq t \leq t_N \\ t_N\ q_i^{\prime}\gamma\ (\frac{N_0-N}{2}) + Nq_i^{\prime}\gamma\ t & t_N \leq t \leq t^* \\ R(1-\ \gamma\ \exp\ (-\lambda(t-t^*)) & t \geq t^* \end{cases}$$

Case 2.

Since $t_1 < t_N$, decline occurs before all wells are drilled, and since $t_N < \theta$, we do not reach the production capacity of the platform. From (7) we see that $Q(t_1) = R(1-\gamma)$.

We now get two "decline" phases, the first from t_1 to t_N, and the second from t_N until production is stopped. From t_1 to t_N we see that the potential is lower than the production capacity of the wells; hence the production is equal to the potential.

$$q(t) = N(t)\ q_i^{\prime}\ (1 - \frac{Q(t)}{R}) \qquad (8)$$

By finding $\frac{d}{dt}\ q(t)$

$$= q_i^{\prime}(1-\ \frac{Q(t)}{R})\ (\frac{1}{\Delta T} - \frac{q_i^{\prime}}{R}\ (N_0 + \frac{t}{\Delta T}\ )^2)$$

we find that $\frac{d}{dt}\ q(t) = 0$ when $Q(t) = R$ and when

$$t_{max} = \sqrt{\frac{R\Delta T}{q_i^{\prime}}} - N_0\ \Delta T$$

If $t_{max} - t_1$ is positive, the production will increase from t_1 to t_{max} and thereafter go into a true decline. If $t_{max} - t_1$ is negative, the production immediately declines.

$$t_{max}-t_1 = \sqrt{\frac{R\Delta T}{q_i^{\prime}}} - \sqrt{\frac{2R\Delta T(1-\gamma)}{q_i^{\prime}\gamma} + (N_0\Delta T)^2}$$

Hence, $t_{max} > t_1$ if and only if

$$\gamma > \frac{2R}{3R-N_0^2\Delta Tq_i^{\prime}}$$

Regardless of the value of γ, however, we see from (8) that for $t_1 \leq t \leq t_N$,

$$\frac{d}{dt}\ Q(t) = q(t)$$

Hence, we solve (8) with the constraint $Q(t_1) = R(1-\gamma)$ and obtain

$$Q(t) = R\ [1 - \gamma\ \exp\ (-\lambda_1(t-t_1) - \frac{\lambda_1}{2N_1\Delta T}\ (t-t_1)^2)] \qquad (9)$$

where $\lambda_1 = \frac{N_1 q_i^{\prime}}{R}$

By differentiating (9) with respect to t, we obtain

$$q(t) = N_1 q_i^{\prime}\gamma\ (1 + \frac{t-t_1}{N_1\Delta T})\ *$$

$$\exp\ (-\lambda_1(t-t_1) - \frac{\lambda_1}{2N_1\Delta T}\ (t-t_1)^2) \qquad (10)$$

Formulae (9) and (10) are valid for $t_1 \leq t \leq t_N$. At t_N, we get

$$Q(t_N) = R[1-\ \gamma\exp\ (-\lambda(t_N-t_1)]$$

$$q(t_N) = Nq_i^{\prime}\gamma\ \exp\ (-\ \frac{\lambda}{2}\ (t_N-t_1)\ (1 + \frac{N_1}{N}))$$

In the following, we shall denote $q(t_N)$ by q_N. At $t = t_N$, drilling stops and we have

$$q(t) = \frac{d}{dt}\ Q(t) = Nq_i^{\prime}\ (1 - \frac{Q(t)}{R})$$

with q_N known. In conclusion, we obtain

$$q(t) = \begin{cases} (N_0 + \frac{t}{\Delta T})\, q_i'\gamma & 0 \le t \le t_1 \\ N_1 q_i'\gamma\, (1 + \frac{t-t_1}{N_1\Delta T}) \exp(-\lambda_1(t-t_1) \\ - \frac{\lambda_1}{2N_1\Delta T}(t-t_1)^2) & t_1 \le t \le t_N \\ q_N \exp(-\lambda(t-t_N)) & t \ge t_N \end{cases}$$

and

$$Q(t) = \begin{cases} q_i'\gamma\, (N_0 + \frac{t}{2\Delta T})t & 0 \le t \le t_1 \\ R[1 - \gamma \exp(-\lambda_1(t-t_1) \\ - \frac{\lambda_1}{2N_1\Delta T}(t-t_1)^2)] & t_1 \le t \le t_N \\ R[1 - \frac{q_N}{Nq_i'} \exp(-\lambda(t-t_N))] & t \ge t_N \end{cases}$$

Case 3

We now reach the plateau in a linear fashion.

$$q(t) = (N_0 + \frac{t}{\Delta T})\, q_i'\gamma \qquad 0 \le t \le \theta$$

$$Q(t) = q_i'\gamma\, (N_0 + \frac{t}{2\Delta T})\, t \qquad 0 \le t \le \theta$$

such that

$$Q(\theta) = q_i'\gamma\, (\frac{N_\theta + N_0}{2})\, \theta$$

where N_θ is the number of wells complete at time θ.

3a. Assume now that decline occurs after t_N. Then production equals q_0 until $t = t^*$. This will happen when

$$q_0 = Nq_i'\, (1 - \frac{Q(t^*)}{R})$$

Hence $\frac{Q(t^*)}{R} = 1 - \frac{q_o}{Nq_i'}$

But $Q(t^*)$ is also given by

$$Q(t^*) = Q(\theta) + q_0(t^*-\theta)$$

and hence we obtain

$$R(1 - \frac{q_0}{Nq_i'}) - Q(\theta) + q_0\theta = q_0 t^*$$

which gives $t^* = \theta - \frac{Q(\theta)}{q_o} + \frac{R}{q_o} - \frac{1}{\lambda}$

After t^*, production equals the potential. Hence we have

$$\frac{d}{dt} Q(t) = Nq_i'\, (1 - \frac{Q(t)}{R})$$

with $Q(t^*)$ known. As a conclusion, we obtain

$$q(t) = \begin{cases} (N_0 + \frac{t}{\Delta T})\, q_i'\gamma & 0 \le t \le \theta \\ q_0 & \theta \le t \le t^* \\ q_0 \exp(-\lambda(t-t^*)) & t \ge t^* \end{cases}$$

$$Q(t) = \begin{cases} q_i'\gamma\, (N_0 + \frac{t}{2\Delta T})\, t & 0 \le t \le \theta \\ Q(\theta) + q_0\, (t-\theta) & \theta \le t \le t^* \\ R[1 - \frac{q_0}{Nq_i'} \exp(-\lambda(t-t^*))] & t \ge t^* \end{cases}$$

3b. $t^* < t_N$. In this case, decline sets in before the last well is completed. At this time, the accumulated production will be

$$Q(t) = Q(\theta) + q_0(t-\theta) \tag{11}$$

At the same time, the potential is

$$N(t)q_i'\, (1 - \frac{Q(t)}{R}) = q_0 \tag{12}$$

$$\text{where } N(t) = (N_0 + \frac{t}{\Delta T}) \tag{13}$$

By substituting (11) and (13) into (12), we obtain a problem in t. The solution t^* shows when the potential equals q_0, and hence when decline occurs. Let

$$B = N_0\Delta T - \theta - \frac{R-Q(\theta)}{q_0}$$

$$C = \frac{\Delta TR}{q_i'} - N_0\, \Delta T\theta - \frac{N_0\Delta T}{q_0}\, (R-Q(\theta))$$

Then t^* solves $t^2 + Bt + C = 0$ and we obtain

$$t^* = \frac{-B + \sqrt{B^2-4C}}{2} \quad \text{and} \quad N^* = t^*/\Delta T + N_0$$

where N^* is the number of wells completed at time t^*. From this we can find

$$Q(t^*) = Q(\theta) + q_0\, (t^*-\theta)$$

In the interval $t^* \le t \le t_N$, we have

$$(N^* + \frac{t-t^*}{\Delta T})\, q_i'\, (1 - \frac{Q(t)}{R}) = \frac{d}{dt} Q(t)$$

with $Q(t^*)$ known.

By solving this differential equation, differentiating with respect to t, and using $q(t^*) = q_0$, we have

$$Q(t) = R[1 - \frac{q_0}{q_i'N^*} \exp(-\lambda^*(t-t^*) - \frac{\lambda^*}{2N^*\Delta T}(t-t^*)^2)]$$

$$q(t) = q_0(1 + \frac{t-t^*}{N^*\Delta T}) \exp(-\lambda^*(t-t^*) - \frac{\lambda^*}{2N^*\Delta T}(t-t^*)^2)$$

where $\lambda^* = \frac{N^*q_i'}{R}$

At $t = t_N$, we find q_N and $Q(t_N)$ from the above formulae. For $t \ge t_N$, we have the differential equation.

$$\frac{d}{dt} Q(t) = Nq_i'\, (1 - \frac{Q(t)}{R})$$

with $Q(t_N)$ known. As a conclusion we have

$$q(t) = \begin{cases} (N_0 + \frac{t}{\Delta T})\, q_i^! \gamma & 0 \le t \le \theta \\ q_0 & \theta \le t \le t^* \\ q_0\, (1 + \frac{t-t^*}{N^*\Delta T})\, \exp\, (-\lambda^*(t-t^*) & \\ - \frac{\lambda^*}{2N^*\Delta T}\, (t-t^*)^2) & t^* \le t \le t_N \\ q_N \exp\, (-\lambda(t-t_N)) & t \ge t_N \end{cases}$$

$$Q(t) = \begin{cases} q_i^! \gamma\, (N_0 + \frac{t}{2\Delta T})\, t & 0 \le t \le \theta \\ Q(\theta) + q_0\, (t-\theta) & \theta \le t \le t^* \\ R[1 - \frac{q_0}{q_i^!\, N^*} \exp\, (-\lambda^*(t-t^*) & \\ - \frac{\lambda^*}{2N^*\Delta T}\, (t-t^*)^2)] & t^* \le t \le t_N \\ R[1- \frac{q_N}{Nq_i^!} \exp\, (-\lambda(t-t_N))] & t \ge t_N \end{cases}$$

Case 4

In this case the potential falls below the production capacity of the wells before all wells are drilled. In this respect Case 4 behaves as Case 2. However, in this case the total number of wells is so large that we might reach the production capacity of the platform, q_0. In order for this to happen, clearly $t_{max} > t_1$. If $t_{max} \le t_1$, Case 4 is equivalent to Case 2. If $t_{max} > t_1$, we must find $q(t_{max})$ as

$$q(t_{max}) = N_1 q_i^! \gamma\, (1 + \frac{t_{max}-t_1}{N_1 \Delta T}) \; *$$

$$\exp\, \{-\lambda_1 (t_{max}-t_1) - \frac{\lambda_1}{2N_1 \Delta T}\, (t_{max}-t_1)^2\}$$

If $q(t_{max}) \le q_0$, Case 4 is again equivalent to Case 2. Hence, the only new case to consider is when $q(t_{max}) > q_0$. In this case the following equation must be solved.

$$q_0 = N_1 q_i^! \gamma\, (1+ \frac{t-t_1}{N_1 \Delta T}) \; *$$

$$\exp\, \{-\lambda_1 (t-t_1) - \frac{\lambda_1}{2N_1 \Delta T}\, (t_1 - t\,)^2\}$$

This will have to be done numerically. This defines the time θ' at which the plateau is reached. (Note that θ' is logically the same as θ, but now the plateau is not reached in a linear fashion).

Following the formulae of Case 2, we can find $Q(\theta')$, but then we are in the situation described in Cases 3a and 3b. Hence, for the case with $q(t_{max}) > q_0$, we conclude that both case 4a and Case 4b behave as Case 2 up till time θ', where $t_1 \le \theta' \le t_N$, and thereafter Case 4a behaves as Case 3a and Case 4b as Case 3 b where θ is replaced by θ'.

A Special Case

There is one situation that is not immediately covered by the previous discussion; that is, when $N_0 \ge \frac{q_0}{\gamma q_i^!}$, i.e., the case when the number of predrilled wells is at least as large as the number needed to reach the platform production capacity q_0. In the calculation this can be discovered by checking if $\theta \le 0$. This brings us into a special version of Case 3. In fact, it can be shown that by letting $\theta = Q(\theta) = 0$, all formulae for Case 3 can be used. In particular, let

$$t^* = \frac{R}{q_0} - \frac{1}{\lambda}$$

If $t^* \ge t_N$, decline will occur after the last well has been drilled and we are in Case 3a. Otherwise, we are in Case 3b, and can find the true t^* by solving an equation of the form:

$$t^2 + Bt + C = 0.$$

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